Name (IN CAPITALS): **Version #1**

Instructor: Dora The Explorer

Math 10550/10350 Exam 3
Apr. 09, 2026.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off (and Put Away) all cellphones, smartwatches and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 23 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)
11.	(•)	(b)	(c)	(d)	(e)
12.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______

13. ______

14. ______

15. ______

16. ______

Total ______

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Instructor: ________________________

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2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

13. ______________

14. ______________

15. ______________

16. ______________

Total ______________

Multiple Choice

1.(7pts) What is the linearization of $f(x) = \sqrt{7+x^2}$ at $x = 3$?

(a) $L(x) = \frac{3}{4}x + \frac{7}{4}$

(b) $L(x) = \frac{3}{4}x - \frac{5}{4}$

(c) $L(x) = \frac{3}{2}x - \frac{1}{2}$

(d) $L(x) = x + 1$

(e) $L(x) = \frac{3}{2}x + 4$

Solution. We use the linearization formula

$$L(x) = f'(a)(x-a) + f(a).$$

Here

$$f(x) = \sqrt{7+x^2} \qquad \text{and} \qquad a = 3.$$

Differentiate:

$$f'(x) = \frac{1}{2}(7+x^2)^{-1/2}(2x) = \frac{x}{\sqrt{7+x^2}}.$$

Now evaluate at $x = 3$:

$$f'(3) = \frac{3}{\sqrt{7+9}} = \frac{3}{4}, \qquad f(3) = \sqrt{16} = 4.$$

Therefore,

$$L(x) = \frac{3}{4}(x-3) + 4 = \frac{3}{4}x + \frac{7}{4}.$$

2.(7pts) Use the linearization of $f(x) = x^{1/3}$ at $x = 8$ to approximate $7^{1/3}$.

(a) 23/12 (b) 25/12 (c) 3/2 (d) 5/6 (e) 7/6

Solution. We use the linearization formula

$$L(x) = f'(a)(x - a) + f(a).$$

Here

$$f(x) = x^{1/3} \qquad \text{and} \qquad a = 8.$$

Differentiate:

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}.$$

Now evaluate at $x = 8$:

$$f'(8) = \frac{1}{3(8)^{2/3}} = \frac{1}{3(4)} = \frac{1}{12}, \qquad f(8) = 8^{1/3} = 2.$$

So the linearization is

$$L(x) = \frac{1}{12}(x - 8) + 2.$$

To approximate $7^{1/3}$, plug in $x = 7$:

$$7^{1/3} \approx L(7) = \frac{1}{12}(7 - 8) + 2 = -\frac{1}{12} + 2 = \frac{23}{12}.$$

3.(7pts) Find the critical values/numbers of the function

$$f(x) = 2x^3 - 15x^2 + 36x + 2026.$$

(a) $x = 2,\ x = 3.$

(b) $x = 0,\ x = -1,\ x = 6.$

(c) $x = 6,\ x = -1$

(d) $x = -2,\ x = -3$

(e) $x = -6,\ x = 1.$

Solution. Critical numbers occur where $f'(x) = 0$ or where $f'(x)$ is undefined.

For

$$f(x) = 2x^3 - 15x^2 + 36x + 2026,$$

we compute

$$f'(x) = 6x^2 - 30x + 36.$$

Factor:

$$f'(x) = 6(x^2 - 5x + 6) = 6(x-2)(x-3).$$

Since $f'(x)$ exists for all x, we only solve

$$6(x-2)(x-3) = 0.$$

Thus the critical numbers are

$$x = 2 \qquad \text{and} \qquad x = 3.$$

4.(7pts) let $v(t)$ denote the velocity of a particle moving along a line at time t secs and let $a(t)$ denote its acceleration at time t secs. Suppose that $a(t) \geq 2$ ft/s^2 for all t, and that $v(0) = 10$ ft/sec. What is the smallest possible velocity of the particle when $t = 3$ sec? (Hint: Use the Mean Value Theorem)

(a) 16 ft/s

(b) 12 ft/s

(c) 6 ft/s

(d) -4 ft/s

(e) 10 ft/s

Solution. Since acceleration is the derivative of velocity, we have

$$a(t) = v'(t).$$

We are given that

$$a(t) \geq 2 \qquad \text{for all } t,$$

and

$$v(0) = 10.$$

Apply the Mean Value Theorem to $v(t)$ on $[0, 3]$. Then for some $c \in (0, 3)$,

$$\frac{v(3) - v(0)}{3 - 0} = v'(c) = a(c).$$

Because $a(c) \geq 2$, we get

$$\frac{v(3) - 10}{3} \geq 2.$$

Multiply by 3:

$$v(3) - 10 \geq 6.$$

So

$$v(3) \geq 16.$$

Therefore, the smallest possible velocity at $t = 3$ is

$$16 \text{ ft/s}.$$

5.(7pts) Let $f(x) = (x-2)^2(3-x)$. Find the critical values/numbers of f and classify the corresponding critical points. Which of the following is true?

(a) f has a local minimum at $x = 2$ and a local maximum at $x = 8/3$.

(b) f has a local maximum at $x = 2$ and a local minimum at $x = 8/3$.

(c) f has a local minimum at $x = 2$ and a local maximum at $x = 3$.

(d) f has a local maximum at $x = 2$ and a local minimum at $x = 3$.

(e) f has a local maximum at $x = 2$ and a local minimum at $x = 4$.

Solution. We are given

$$f(x) = (x-2)^2(3-x).$$

To find the critical numbers, compute the derivative:

$$f'(x) = 2(x-2)(3-x) + (x-2)^2(-1).$$

Factor out $(x-2)$:

$$f'(x) = (x-2)\big(2(3-x) - (x-2)\big).$$

Simplify:

$$f'(x) = (x-2)(6-2x-x+2) = (x-2)(8-3x).$$

Critical numbers occur when $f'(x) = 0$ or when $f'(x)$ is undefined. Since $f'(x)$ is defined for all x, solve

$$(x-2)(8-3x) = 0.$$

So the critical numbers are

$$x = 2 \qquad \text{and} \qquad x = \frac{8}{3}.$$

Now classify them using the sign of $f'(x)$.

If $x < 2$, then

$$(x-2) < 0 \quad \text{and} \quad (8-3x) > 0,$$

so $f'(x) < 0$.

If $2 < x < \frac{8}{3}$, then

$$(x-2) > 0 \quad \text{and} \quad (8-3x) > 0,$$

so $f'(x) > 0$.

If $x > \frac{8}{3}$, then

$$(x-2) > 0 \quad \text{and} \quad (8-3x) < 0,$$

so $f'(x) < 0$.

Thus f changes from decreasing to increasing at $x = 2$, so $x = 2$ is a local minimum.

Also, f changes from increasing to decreasing at $x = \frac{8}{3}$, so $x = \frac{8}{3}$ is a local maximum.

6.(7pts) Let $f(x) = x^4 - 8x^3 + 18x^2 + 22x + 355640$. Which of the following is true?

(a) The graph of f is concave up on the intervals $(-\infty, 1)$ and $(3, \infty)$.

(b) The graph of f is concave up on the interval $(1, 3)$.

(c) The graph of f is concave up on the intervals $(0, 2)$ and $(3, \infty)$.

(d) The graph of f is concave up on the intervals $(-\infty, 0)$ and $(1, 3)$.

(e) The graph of f is concave down everywhere on its domain.

Solution. To determine where the graph is concave up, compute the second derivative.

For

$$f(x) = x^4 - 8x^3 + 18x^2 + 22x + 355640,$$

the first derivative is

$$f'(x) = 4x^3 - 24x^2 + 36x + 22,$$

and the second derivative is

$$f''(x) = 12x^2 - 48x + 36.$$

Factor:

$$f''(x) = 12(x^2 - 4x + 3) = 12(x - 1)(x - 3).$$

The graph is concave up where

$$f''(x) > 0.$$

Since $(x-1)(x-3) > 0$ when both factors are positive or both are negative, this happens when

$$x < 1 \qquad \text{or} \qquad x > 3.$$

Therefore, the graph is concave up on

$$(-\infty, 1) \qquad \text{and} \qquad (3, \infty).$$

7.(7pts) Find the x co-ordinates of the inflection points of the curve

$$f(x) = \frac{x^2}{2\sqrt{2}} + \cos(x)$$

on the interval $0 \leq x \leq 2\pi$?

(a) $\frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\frac{3\pi}{4}, \frac{5\pi}{4}$

(c) $\frac{\pi}{3}, \frac{5\pi}{3}$

(d) $\frac{2\pi}{3}, \frac{4\pi}{3}$

(e) $\frac{\pi}{2}, \frac{3\pi}{2}$

Solution. Inflection points occur where the second derivative changes sign.

For

$$f(x) = \frac{x^2}{2\sqrt{2}} + \cos(x),$$

we compute

$$f'(x) = \frac{x}{\sqrt{2}} - \sin(x)$$

and

$$f''(x) = \frac{1}{\sqrt{2}} - \cos(x).$$

Set the second derivative equal to zero:

$$\frac{1}{\sqrt{2}} - \cos(x) = 0.$$

So

$$\cos(x) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

On the interval $0 \leq x \leq 2\pi$, this happens at

$$x = \frac{\pi}{4} \qquad \text{and} \qquad x = \frac{7\pi}{4}.$$

At both of these values, $f''(x)$ changes sign, so both are inflection points.
Therefore, the x-coordinates of the inflection points are

$$\frac{\pi}{4} \qquad \text{and} \qquad \frac{7\pi}{4}.$$

8.(7pts) Find

$$\lim_{x\to 0^+} x^x$$

(a) 1 (b) 0 (c) $+\infty$ (d) $-\infty$ (e) e

Solution. We want to find

$$\lim_{x\to 0^+} x^x.$$

Let

$$y = x^x.$$

Then

$$\ln(y) = \ln(x^x) = x\ln(x).$$

So it is enough to compute

$$\lim_{x\to 0^+} x\ln(x).$$

Rewrite this as

$$x\ln(x) = \frac{\ln(x)}{1/x}.$$

Now apply L'Hospital's Rule:

$$\lim_{x\to 0^+} \frac{\ln(x)}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} (-x) = 0.$$

Thus

$$\lim_{x\to 0^+} \ln(y) = 0.$$

Undoing the initial ln gives

$$\lim_{x\to 0^+} e^{ln(y)} = e^0 = 1.$$

Hence

$$\lim_{x\to 0^+} x^x = 1.$$

9.(7pts) Find

$$\lim_{x\to+\infty} \frac{1-e^x}{x^2+x+1}$$

(a) $-\infty$ (b) $+\infty$ (c) 0 (d) e (e) 2

Solution. We want to compute

$$\lim_{x\to+\infty} \frac{1-e^x}{x^2+x+1}.$$

As $x \to +\infty$, the numerator goes to $-\infty$ and the denominator goes to $+\infty$, so we may apply L'Hospital's Rule.

Differentiate the numerator and denominator:

$$\lim_{x\to+\infty} \frac{1-e^x}{x^2+x+1} = \lim_{x\to+\infty} \frac{-e^x}{2x+1}.$$

This is still an indeterminate form, so apply L'Hospital's Rule again:

$$\lim_{x\to+\infty} \frac{-e^x}{2x+1} = \lim_{x\to+\infty} \frac{-e^x}{2}.$$

Since $e^x \to +\infty$, we get

$$\frac{-e^x}{2} \to -\infty.$$

Therefore,

$$\lim_{x\to+\infty} \frac{1-e^x}{x^2+x+1} = -\infty.$$

10.(7pts) Which of the following gives the most general antiderivative of $f(x)$, where

$$f(x) = \frac{2}{x^3} - \frac{1}{x^2}$$

(a) $\dfrac{1}{x} - \dfrac{1}{x^2} + C$

(b) $\dfrac{-6}{x^4} + \dfrac{2}{x^3} + C$

(c) $\dfrac{-2}{3x^2} + \dfrac{1}{2x} + C$

(d) $2\ln(x^3) - \ln(x^2)$

(e) $\ln(x^2) - 2\ln(x^3)$

Solution. We want the most general antiderivative of

$$f(x) = \frac{2}{x^3} - \frac{1}{x^2}.$$

Rewrite with exponents:

$$f(x) = 2x^{-3} - x^{-2}.$$

Now anti-derive term by term:

$$F(x) = 2\left(\frac{x^{-2}}{-2}\right) - \left(\frac{x^{-1}}{-1}\right) = -x^{-2} + x^{-1}.$$

Add $+C$,

$$F(x) = -x^{-2} + x^{-1} + C.$$

Rewrite with positive exponents:

$$F(x) = \frac{1}{x} - \frac{1}{x^2} + C.$$

11.(7pts) If $f''(x) = 3$ with $f'(1) = 4$ and $f(2) = 10$, find $f(0)$.

(a) 2 (b) 0 (c) 1 (d) 3 (e) 4

Solution. We are given

$$f''(x) = 3, \qquad f'(1) = 4, \qquad f(2) = 10.$$

To find $f(0)$, integrate twice.
First integrate $f''(x) = 3$:

$$f'(x) = 3x + C.$$

Use $f'(1) = 4$:

$$4 = 3(1) + C,$$

so

$$C = 1.$$

Thus

$$f'(x) = 3x + 1.$$

Now integrate again:

$$f(x) = \frac{3}{2}x^2 + x + D.$$

Use $f(2) = 10$:

$$10 = \frac{3}{2}(2)^2 + 2 + D = 6 + 2 + D = 8 + D.$$

So

$$D = 2.$$

Hence

$$f(x) = \frac{3}{2}x^2 + x + 2.$$

Finally,

$$f(0) = 2.$$

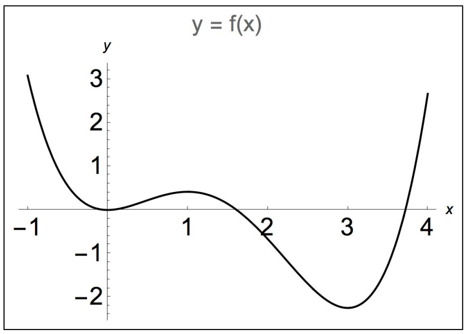
Initials: ____________

12.(7pts) Let f be a function of x. The table below shows whether the functions $f'(x)$ and $f''(x)$ are positive, negative or have value 0 at each of the given values of x.

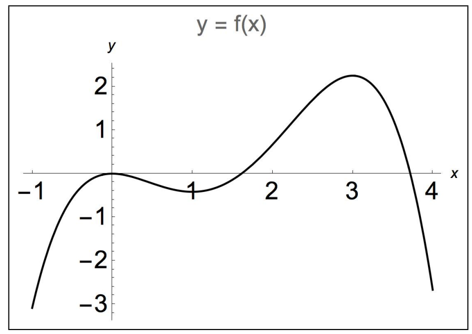
x	0	1	3
$f'(x)$	$= 0$	$= 0$	$= 0$
$f''(x)$	> 0	< 0	> 0

Which of the graphs shown below is a feasible graph of $f(x)$?

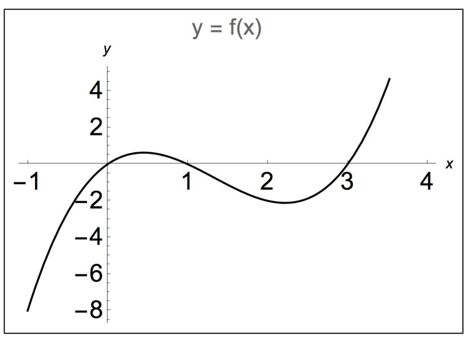
(a)



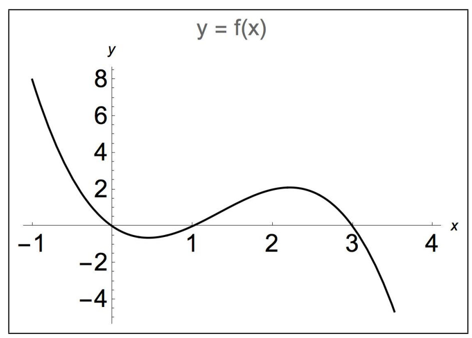
(b)



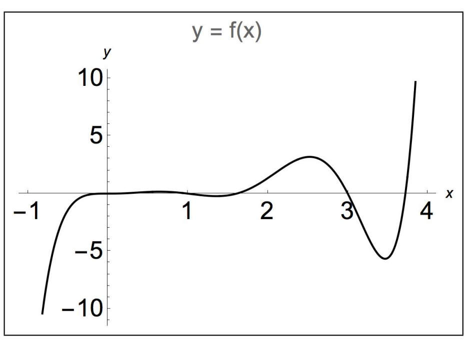
(c)



(d)



(e)



Solution. The table tells us that

$$f'(0) = 0, \qquad f'(1) = 0, \qquad f'(3) = 0.$$

So the graph has horizontal tangents at $x = 0$, $x = 1$, and $x = 3$.

The table also gives the sign of $f''(x)$:

$$f''(0) > 0, \qquad f''(1) < 0, \qquad f''(3) > 0.$$

That means:

$$x = 0 \text{ is a local minimum,}$$
$$x = 1 \text{ is a local maximum,}$$
$$x = 3 \text{ is a local minimum.}$$

So we need the graph with a local minimum at $x = 0$, a local maximum at $x = 1$, and a local minimum at $x = 3$.

Among the five choices, the first graph is the only one that matches these conditions.

Partial Credit

For full credit on partial credit problems, make sure you justify your answers.
You are not expected to simplify the arithmetic in your answers.

13.(12pts) Consider the function

$$f(x) = x(x-3)^2$$

Hint: It's best not to expand $f(x)$ before you differentiate

(a) Calculate $f'(x)$.

(b) Find the critical numbers of $f(x)$?

(c) Find the maximum value of $f(x)$ on the interval $0 \le x \le 5$ and briefly explain your work.

Solution. **(a)** We are given

$$f(x) = x(x-3)^2.$$

Differentiate using the product rule:

$$f'(x) = 1 \cdot (x-3)^2 + x \cdot 2(x-3).$$

Factor out $(x-3)$:

$$f'(x) = (x-3)\big((x-3) + 2x\big).$$

Simplify:

$$f'(x) = (x-3)(3x-3) = 3(x-3)(x-1).$$

(b) Critical numbers occur when $f'(x) = 0$ or when $f'(x)$ is undefined.
Since $f'(x)$ exists for all x, solve

$$3(x-3)(x-1) = 0.$$

So the critical numbers are

$$x = 1 \qquad \text{and} \qquad x = 3.$$

(c) To find the maximum value on the interval $0 \leq x \leq 5$, check the endpoints and the critical numbers in the interval.

Evaluate:

$$f(0) = 0(0-3)^2 = 0,$$
$$f(1) = 1(1-3)^2 = 4,$$
$$f(3) = 3(3-3)^2 = 0,$$
$$f(5) = 5(5-3)^2 = 20.$$

Comparing these values, the largest is

$$20.$$

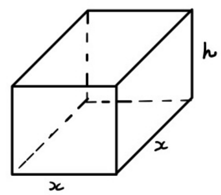
Therefore, the maximum value of $f(x)$ on $[0, 5]$ is

$$20,$$

and it occurs at

$$x = 5.$$

14.(12pts) If 4 ft^2 of cardboard is available to make a box with **a square base and an open top**. Find the volume of the box with the largest possible volume that can be made from the cardboard.



Volume: ________ft^3

Solution. Let x be the side length of the square base and let h be the height of the box.
Since the box has no top, the surface area is

$$x^2 + 4xh = 4.$$

Solve for h:

$$4xh = 4 - x^2, \qquad h = \frac{4 - x^2}{4x}.$$

The volume is

$$V = x^2 h.$$

Substitute the expression for h:

$$V(x) = x^2 \left(\frac{4 - x^2}{4x} \right).$$

Simplify:

$$V(x) = \frac{x(4 - x^2)}{4} = x - \frac{x^3}{4}.$$

Differentiate:

$$V'(x) = 1 - \frac{3x^2}{4}.$$

Set the derivative equal to zero:

$$1 - \frac{3x^2}{4} = 0.$$

So

$$\frac{3x^2}{4} = 1, \qquad x^2 = \frac{4}{3}, \qquad x = \frac{2}{\sqrt{3}}.$$

We use the positive value because a side length must be positive.
Now compute the second derivative:

$$V''(x) = -\frac{3x}{2}.$$

We have $x > 0$, and $x^2 \leq 4$ since the base has area ≤ 4. We have $V''(x) < 0$, so this critical point gives a maximum volume because it is the unique critical point in the interval $[0, 2]$. (One could also apply the Extreme Value Theorem here to verify that the global max occurs at this Critical point.)
Now find the height:

$$h = \frac{4 - \left(\frac{2}{\sqrt{3}} \right)^2}{4 \left(\frac{2}{\sqrt{3}} \right)} = \frac{4 - \frac{4}{3}}{\frac{8}{\sqrt{3}}} = \frac{\frac{8}{3}}{\frac{8}{\sqrt{3}}} = \frac{\sqrt{3}}{3}.$$

Finally, the maximum volume is

$$V = x^2 h = \left(\frac{2}{\sqrt{3}} \right)^2 \left(\frac{\sqrt{3}}{3} \right) = \frac{4}{3} \cdot \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{9}.$$

So the box with largest volume has

$$x = \frac{2}{\sqrt{3}} \text{ ft}, \qquad h = \frac{\sqrt{3}}{3} \text{ ft},$$

and maximum volume

$$\frac{4\sqrt{3}}{9} \text{ ft}^3.$$

True-False.

15.(4pts) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) If c is a critical number of $f(x)$, then the point $(c, f(c))$ must be a local maximum or a local minimum on the graph of $f(x)$.

TRUE FALSE

(b)(1 pt. No Partial credit) One can apply L'Hospital's rule to find the limit

$$\lim_{x \to 0} \frac{\cos x + 1}{\sin x}$$

TRUE FALSE

(c)(1 pt. No Partial credit) $f(x) = \cos(x)$ has a local maximum at $x = 0$.

TRUE FALSE

(d)(1 pt. No Partial credit) If $f'(x) = 0$ for all values of x, then $f(x) = k$, where k is a constant, for all values of x.

TRUE FALSE

Solution. (a) FALSE.

A critical number is a number where $f'(x) = 0$ or where $f'(x)$ is undefined.

A critical number does not have to give a local maximum or local minimum. For example,

$$f(x) = x^3$$

has a critical number at $x = 0$, but $x = 0$ is neither a local maximum nor a local minimum.

(b) FALSE.

L'Hospital's Rule applies only to indeterminate forms such as

$$\frac{0}{0} \qquad \text{or} \qquad \frac{\infty}{\infty}.$$

Here,

$$\cos(0) + 1 = 2 \qquad \text{and} \qquad \sin(0) = 0,$$

so the expression is not an indeterminate form. Therefore, L'Hospital's Rule does not apply.

(c) TRUE.

The function

$$f(x) = \cos(x)$$

has value 1 at $x = 0$, and nearby values are less than or equal to 1. Therefore, $x = 0$ is a local maximum.

(d) TRUE.

If

$$f'(x) = 0 \qquad \text{for all } x,$$

then the slope is zero everywhere, so the function is constant. Thus

$$f(x) = k$$

for some constant k.

16.(2pts) You will be awarded these two points if you write your name in CAPITALS on the front page and you mark your answers on the front page with an X through your answer choice like so: ~~(a)~~ (<u>not</u> an O around your answer choice) .

The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$a^x = e^{x\ln(a)}$$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Initials: ____________

ROUGH WORK